Dynamic Performance Comparison of PD and PD-Fuzzy Logic Controllers for Inverted Pendulum System

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ABSTRACT

Inverted pendulum system is a well-known conventional tool used to test the success and the ability of the various control methods. Inverted pendulum system consists of mostly an inverted pendulum and a movable cart that it is mounted on. Certainly, this system has a highly complex nonlinear structure that the simple control methods could not cope with it. Main purpose of the control is to keep the pendulum balanced and stable in the upright position in that it is inherently unstable, while keeping cart position in a determined place on the surface. In this study, separately, PD and PD-Fuzzy control methods have been used to control the inverted pendulum system.

Keywords: PD controller, PD-fuzzy controller, Inverted Pendulum

1 INTRODUCTION

Inverted pendulum system is inherently a multi-variable, severely non-linear and unstable system. Therefore, it must be used a resourceful and talented control method to make it stable. In general, there are big difficulties in the control of the complex nonlinear systems why they have not good mathematical models obtained by classical methods based on differential equations. This problem has been exceeded in a great degree by using Artificial Intelligence based adaptive control methods.

As known, the best-known examples of the applications of Artificial Intelligence are those based on fuzzy logic and artificial neural networks. In recent years, a large amount of control and modeling methods have been developed based on fuzzy logic and artificial neural networks and used for the controlling of inverted pendulum and other complex systems [1-7].

In this study, first, a nonlinear mathematical model of the inverted pendulum system has been obtained in the form of differential equations, then by considering this complex model as if real plant, simulation studies have been performed using conventional PD controller and hybrid PD-fuzzy controller for angular position and cart position control in Matlab-Simulink environment.

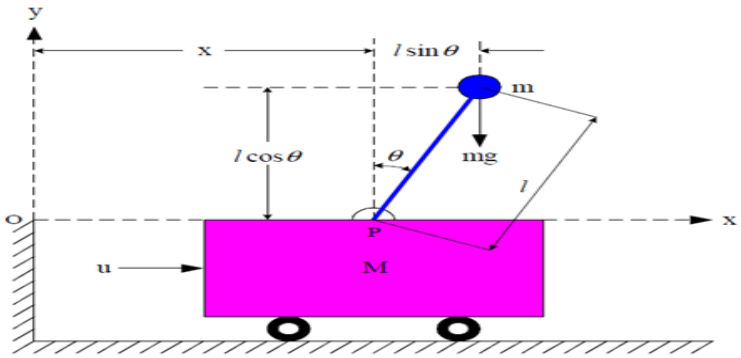
2 MATHEMATICAL MODEL OF INVERTED PENDULUM SYSTEM

Dynamics of inverted pendulum are basic to tasks involving maintenance of balance such as walking and the control of launching a rocket, which must be balanced on its own thrust vector by rotating the engine. As it can be seen in figure 1, inverted pendulum system has its length (l), mass (m) accepted intensified in bar endpoints, mounted on a cart with mass of M at point P. The angle with vertical position rod's passing through the point P and the distance from a reference point on the horizontal axis are represented by Θ and x. Rod movement is constrained to the x-y plane and the rod mounted on cart is able to move only along the x axis [8].

Gravitational center of rod's mass can be expressed in x-y coordinate plane as below; equation (1) and (2):

$$x_G = x + l\sin\theta \tag{1}$$

$$y_G = l\cos\theta \tag{2}$$



Şekil 1: Inverted Pendulum System

When Newton's second law of motion is applied in the x direction, in equation (3) differential equation is obtained.

$$M\frac{d^2x}{dt^2} + m\frac{d^2x_G}{dt^2} = u \tag{3}$$

If differential equation in equation (1) is substituted equation (3), equation (4) is obtained.

$$(\mathrm{M}+\mathrm{m})\ddot{x} - ml(\sin\theta)\dot{\theta}^2 + ml(\cos\theta)\ddot{\theta} = u \tag{4}$$

Secondly, when newton's second law of motion is applied to the motion of mass (m) around the point P, differential equation in equation (5) is obtained.

$$m\frac{d^2x_G}{dt^2}l\cos\theta - m\frac{d^2y_G}{dt^2}l\sin\theta = mgl\sin\theta \tag{5}$$

If Equation (1) and equation (2) above is performed, equation (6) is obtained. The value of the inverted pendulum system is given in table 1.

Table I Inverted pendulum system values

Definition	Symbol	Value	Unit
Mass of cart	M	2.4	kg
Mass of inverted pendulum	m	0.23	kg
Length of inverted pendulum rod	l	0.36	m
Acceleration of gravity	g	9.8	m/sn^2
Runway length	L	0.5	m

3 FUZZY LOGIC CONTROL

Since its introduction by Mamdani in 1974, fuzzy logic control has been applied in a variety of ways, and therefore, many different types of fuzzy logic control have been introduced. However, most of the fuzzy logic controllers in use have some common characteristics. A block diagram showing the basic configuration of an fuzzy logic controller is shown in figure 2[9]. Fuzzy logic controller, with general structure, makes up of four basic components as fuzzification, knowledge base, inference and defuzzification.

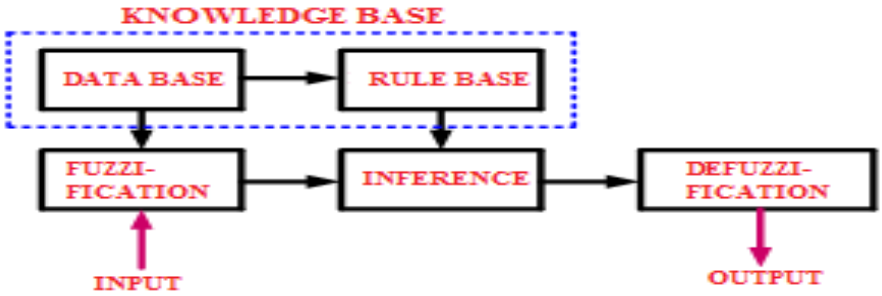


Figure 2: General structure of the fuzzy logic controller

Fuzzification converts a crisp value of a process variable into a fuzzy set in order to make it compatible with the fuzzy set representation of the process state variable in the rule-antecedent. Fuzzification plays an important role in dealing with uncertain information which might be objective in nature. In fuzzy control applications, the observed data are usually crisp. Since the data manipulation in a fuzzy logic control is based on fuzzy set theory, fuzzification is necessary during an earlier stage. The knowledge base of a fuzzy logic controller contains a data base and a rule base. The data base provides necessary definitions for linguistic control rules and fuzzy data manipulation, and the rule base characterizes the control goals and policies by means of a set of control rules. Fuzzy values received fuzzification unit are produced fuzzy results implementing on rule base in inference unit. The output decision of a fuzzy logic controller is a fuzzy value and is represented by a membership function to precise or crisp quantity. A defuzzification strategy is aimed at producing a non-fuzzy control action that represents the possibility distribution of an inferred fuzzy control action. There are so many inference methods and approaches to the design of fuzzy logic controller, but the most popular and in use are two inference methods. The two inference methods are; Mamdani method and Sugeno method. In this study, mamdani inference method has been used[3]. Input and output variables in fuzzy logic controller blur process are converted into a symbolic expression. In this study, the uncertainty is characterized with seven linguistic variables.These are NB (Negative Big), NM (negative medium), NS (Negative Small), ZR (Zero), PS (Positive Small), PM (Positive Medium), PB (Positive Big).The choice of the membership functions for each input to the system may be triangular, trapezoidal, sinusoidal, cauchy, bell, sigmoid, gaussian types. Triangular membership functions are used in

this study. The position of the cart and angular position of inverted pendulum used for triangular shaped membership function belonging to error, error change and output is shown in figure 3-5.

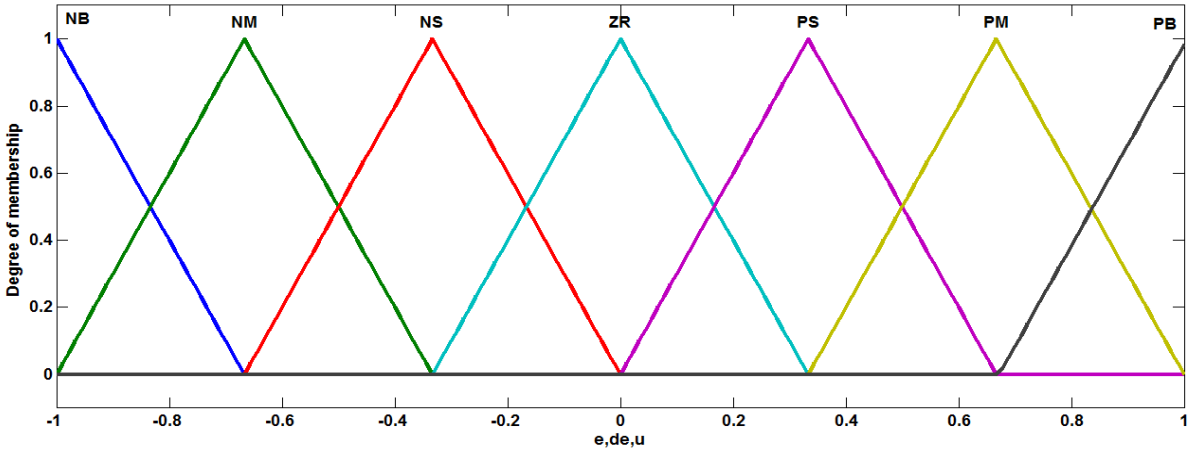


Figure 3: The error, error change, output membership functions for the position control of cart.

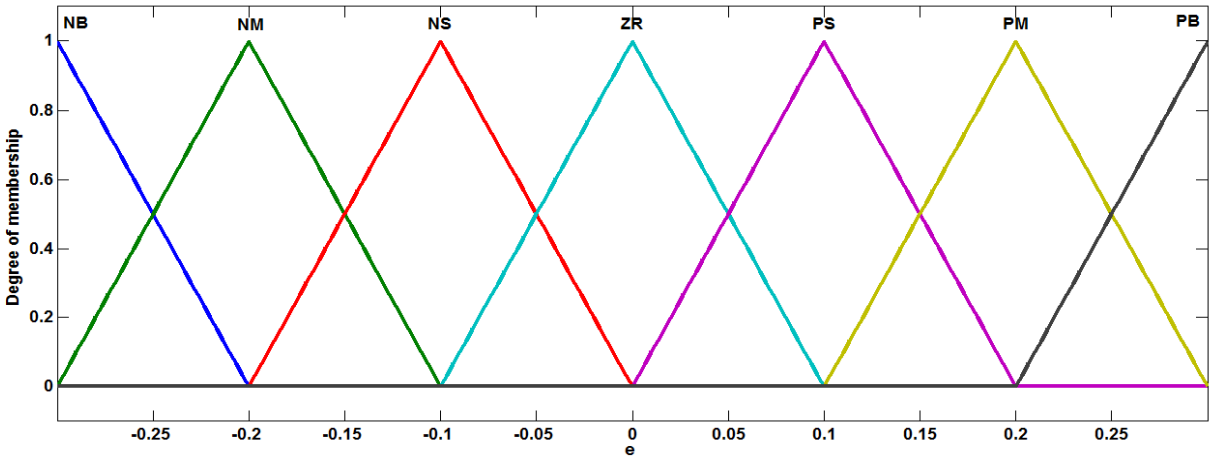


Figure 4: The error membership functions for the angular position control of inverted pendulum.

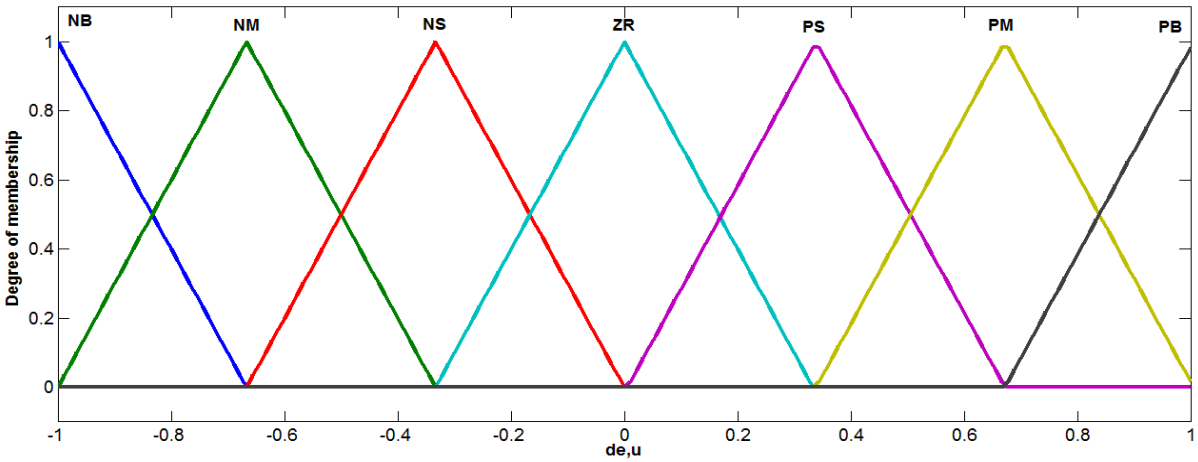


Figure 5 :The error change and output membership functions for the angular position control of inverted pendulum.

The angular position control of the inverted pendulum and position control of the cart used in the rule table is given in table 2. In the controller input fuzzy inference unit ,its relationship with benefiting from specialist knowledge is provided by the rule set. In this study forty-nine rules were created by using all linguistic variables.

Table 2 Rule Table

u		***de***						
		NB	NM	NS	ZR	PS	PM	PB
e	NB	NB	NB	NB	NB	NM	NS	ZR
	NM	NB	NB	NB	NM	NS	ZR	PS
	NS	NB	NB	NM	NS	ZR	PS	PM
	ZR	NB	NM	NS	ZR	PS	PM	PB
	PS	NM	NS	ZR	PS	PM	PB	PB
	PM	NS	ZR	PS	PM	PB	PB	PB
	PB	ZR	PS	PM	PB	PB	PB	PB

4 PD CONTROL

In a closed loop control system, the task of control elements size measuring element output fed back through to compare with the size of the input and comparison of the error value may arise from the structure and depending on its control actions is to generate an appropriate command or control signal. There are three fundamental control effects used for classical control systems. They are proportion (P) effect, integral (I) effect and derivative (D) effect. The general structure of a closed loop fuzzy control system is provided in figure 6.

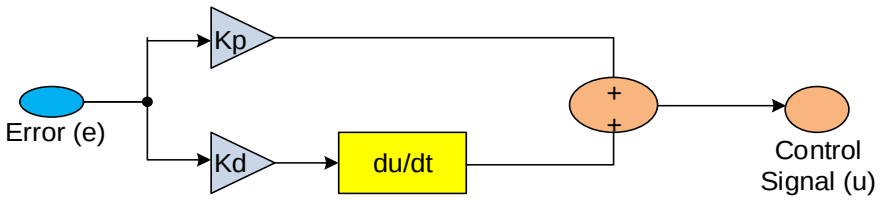


Figure 6: The general structure of PD controller

Working in various supervision bodies (P, PI, PD, PID) are created this combination of one or more underlying control effects with used appropriately. PD controller in the control system is used to minimize the system rise time, overshoot and instability. The output equations for PD controller is given in equation 6.

$$u(t) = K_p.e(t) + K_d.\frac{d}{dt}e(t) \tag{6}$$

5 PD-FUZZY CONTROL

The fuzzy controllers are also designed different structure as in conventional controllers as in the form of hybrid fuzzy controllers PD fuzzy, PI fuzzy, PID fuzzy. PD-fuzzy control system was created with reference to classical PD control system and is a fuzzy control system which has two- input single-output. A PD linear controller is composed of proportional and derivative gain factor [10-14]. PD-fuzzy controller structure is shown in the figure 7.

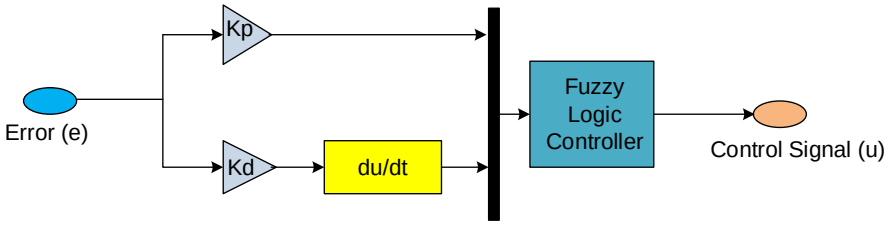


Figure 7: Structure of PD-fuzzy controller

6 SIMULATION STUDIES

Nonlinear model of inverted pendulum system is modelled in MATLAB/Simulink. PD and PD fuzzy controller are implemented to modelled inverted pendulum system. The tuning of controller parameters(K_P and K_d) is done using trial and error method and observing the responses of controllers to be the optimal Proportional gain value (K_P) and derivative gain value (K_d) for PD controller in angular position control of inverted pendulum are set to K_p=35, K_d=8.Proportional gain value (K_p) and derivative gain value (K_d) for PD controller in position control of cart are set to K_p=1.25, K_d=3.6.Proportional gain value (K_P) and derivative gain value (K_d) for PD-fuzzy controller in angular position control of inverted pendulum are set to K_p=8, K_d=5.Proportional gain value (Kp) and derivative gain value (Kd) for PD-fuzzy controller in position control of cart are set to Kp=1.6, Kd=3.Control signal has been applied to inverted pendulum system by limiting in the range of u=[-10,10]. Initial angular position of the inverted pendulum has been taken 2 degree. Initial position of the cart has been taken 0 m. The reference angular position value of inverted pendulum is taken 0 degree, the reference position value of cart is taken 0.1m. 0.25N distorting amplitude was implemented to the system in 15. seconds. Disruptive impact mark designed for system has been given in figure 8.

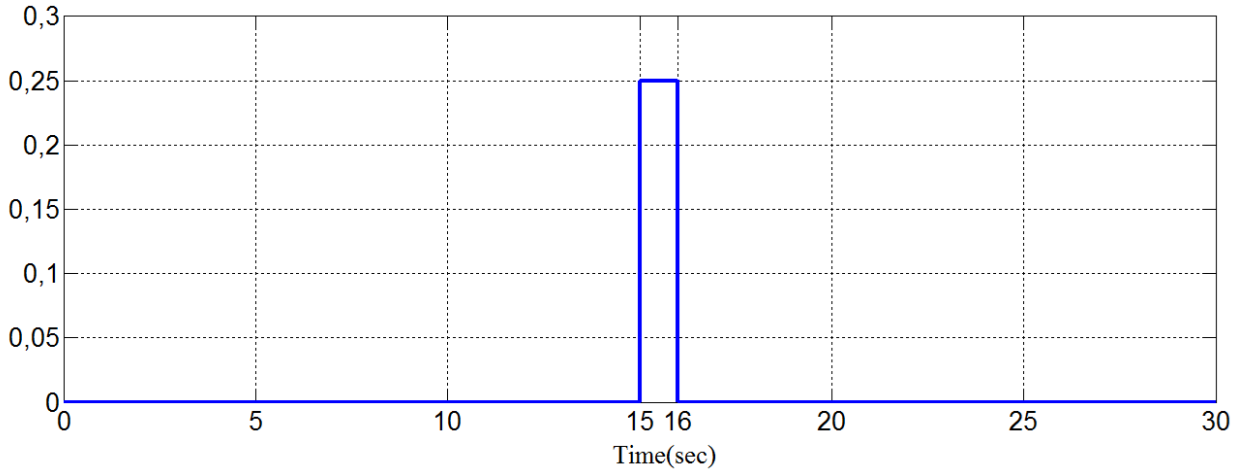


Figure 8: Disruptive impact mark designed for inverted pendulum system

PD-fuzzy controller model designed in MATLAB/Simulink for inverted pendulum is given in figure 9.

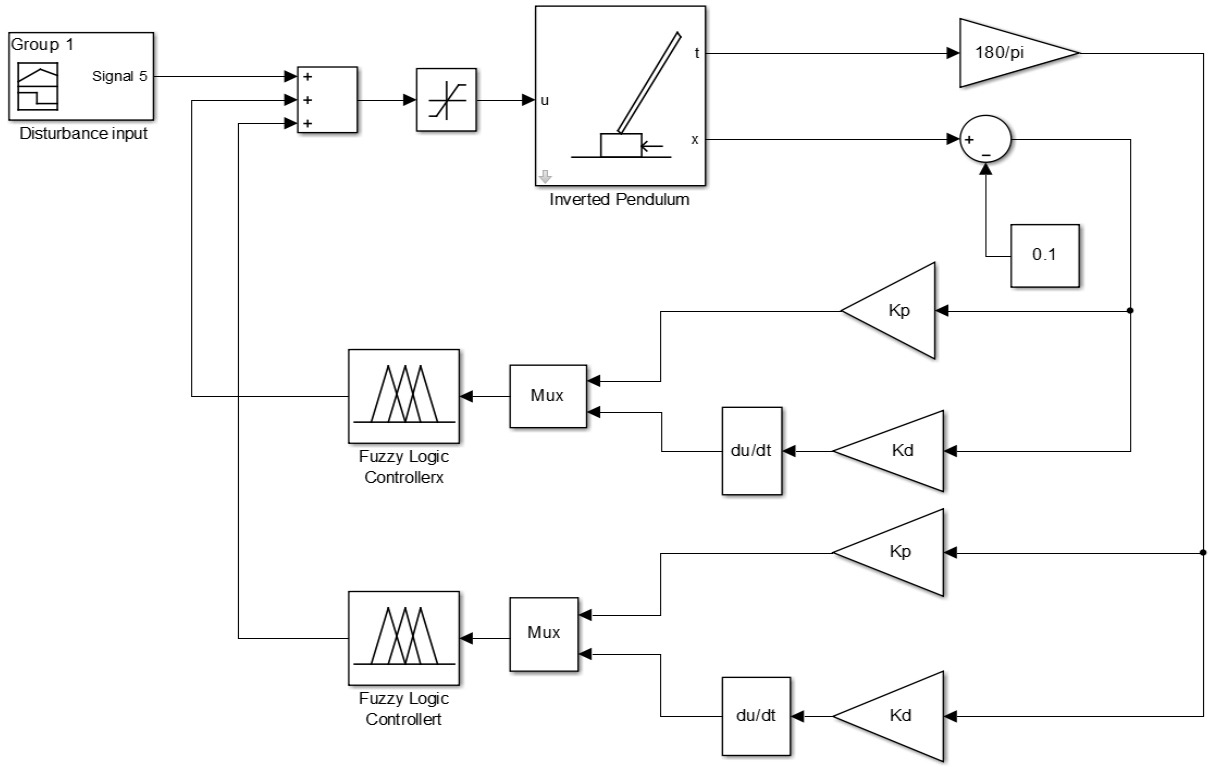


Figure 9:PD fuzzy controller model designed in Matlab/Simulink for inverted pendulum

PD controller model designed in MATLAB/Simulink for inverted pendulum is given in figure 10.

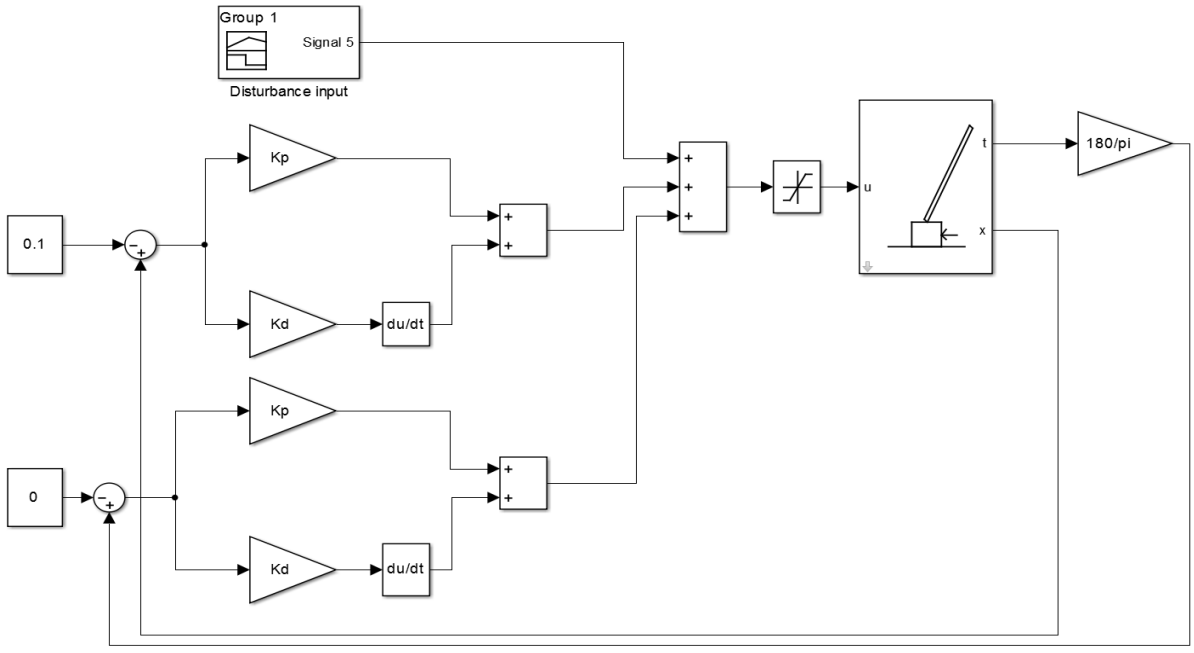


Figure 10: PD controller model designed in Matlab/Simulink for inverted pendulum

Simulation results in MATLAB/Simulink belonging to PD controller and PD-fuzzy controller for angular position of inverted pendulum are given in figure 11. Simulation results belonging to PD and PD-fuzzy controller for position control of the cart are given in figure 12.

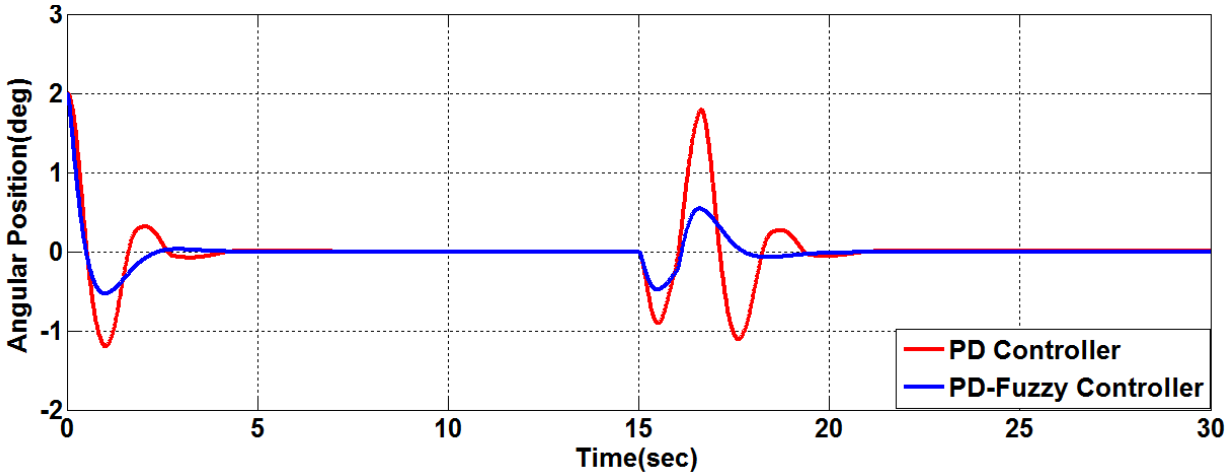


Figure 11: Angular position control of the inverted pendulum with PD controller and PD fuzzy controller

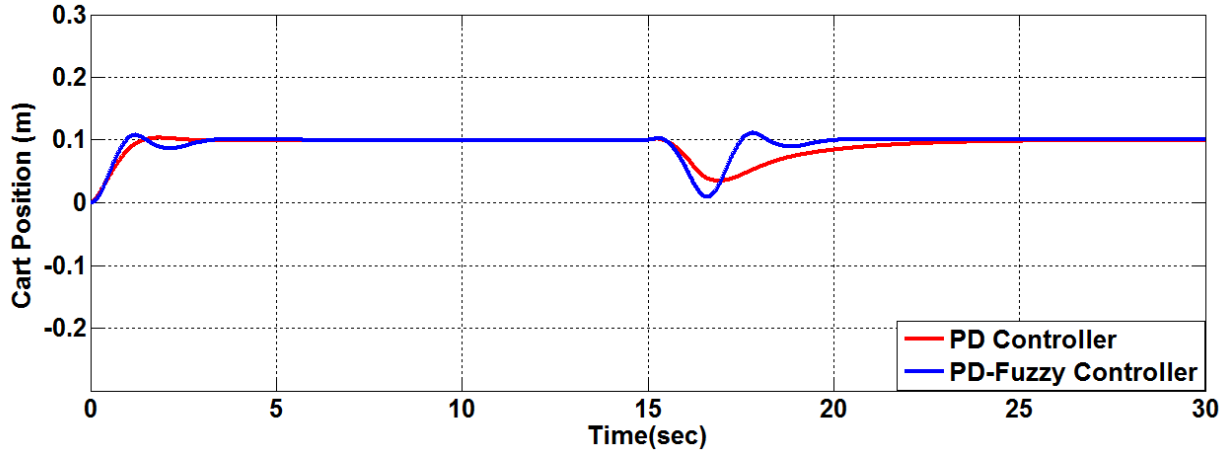


Figure 12: Position control of cart with PD controller and PD fuzzy controller

In the PD fuzzy controller design for controlling inverted pendulum angular and cart position, when the nine and twenty-five rule based controller is applied, steady state error is seen that not completely eliminated. Therefore, it is understood that the necessary of using fuzzy controller that has minimum forty-nine rule for controlling inverted pendulum. In the choosing membership function, the triangular membership function is selected due to narrow peak point, high slope and fasting response to error change. It is aimed that the system work is stable by improving gain adjusting factor (K_p and K_d) in PD fuzzy controller with error and error change membership function interval. In PD controller design, K_p and K_d parameter is determined with trying studies by analysing system response. The system response of inverted pendulum for angular position control using PD and PD-fuzzy control method is shown in figure.11.It is observed that PD-fuzzy control schemes the pendulum stabilizes in vertically upright position quickly and smoothly in approx. 5 seconds after a minor undershoot before and after disturbance. But PD control schemes the pendulum stabilizes in vertically upright position after a major undershoot and a major overshoot before and after disturbance. The system response of inverted pendulum for cart position control using PD and PD-fuzzy control method is shown in figure 12.The cart position control with PD-fuzzy

controller, x reaches desired position of 0.1 (m) more quickly than PD controller in approx. 1 seconds before disturbance. After disturbance is applied, PD-fuzzy controller reaches desired position (0.1m) faster than PD controller.

7 CONCLUSIONS

Inverted pendulum system is a well-known conventional tool used to test the success and the ability of the various control methods. The main objective of this work is comparatively to demonstrate the successes of conventional and fuzzy control algorithms with the help of inverted pendulum system. PD type classical controller has been preferred for conventional control. The advanced control method considered consists of PD and fuzzy algorithms. PD-fuzzy control is a hybrid control method that combines the best features of classical and fuzzy control algorithms. Both control methods before mentioned serves for keeping of the pendulum balanced and stable in the upright position in that it is inherently unstable, while keeping cart position in a determined place on the surface. Simulation results, figures 11 and 12, presents angular and cart positions for the inverted pendulum system under the effects of disturbances. As clearly shown, PD-Fuzzy control is more successful than classical PD control considering basic performance criteria; suppression of disturbances, response speed,overshoot and settling time.

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