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Oral Presentation

Dynamic Performance Comparison of LQR and LQI Controllers on Buck Converter

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Abstract

DC-DC converters are electronics elements which change voltage level based on switching. Unlike linear circuit topologies, modeling and control of DC-DC converter circuits are fairly difficult due to their switched structure. The main problems are to model semiconductor switches, to identify various circuit structures and model the converter following an analysis of the states. State-space model and system modeling are effectively used in converter designs. The required transfer function is obtained thanks to a small signal analysis of the system modeled using state-space model at a certain operating point. In optimal controlling methods, the controller parameters based on the transfer function of the controlled system are minimized using a known error criterion. Linear Quadratic Regulator (LQR) controller is one of the most widely used optimal control methods. Linear Quadratic Integral (LQI) controller is obtained by adding integral control to the LQR in order to increase the performance of control system.
In this study, the circuit model of an optimal control system was designed in Matlab/Simulink by obtaining the state-space of a buck DC-DC converter using small signal model. The reference voltage and output load of the buck DC-DC converter were varied to compare dynamic performances of LQR and LQI controllers.

Keywords: Optimal Control, Buck Converter, LQR and LQI Controllers.

1.Introduction

In today's world, rapid industrial development requires the use of power electronics circuits. A number of converters have been so far proposed in parallel with the developments in the field of power electronics. Various types of DC-DC converter circuits exist. While some converters are used to decrease or increase voltage, some types are used to both increase and decrease voltage (Calderon ve ark., 2006 ; Dekker ve ark.,1984) Buck DC-DC converter is a power electronics circuit that converts input voltage level into a voltage level lower than the input voltage level. The stability of a control system designed for a buck DC-DC converter depends on dynamic modeling. The linear model of a buck DC-DC converter is obtained thanks to average state-space method using circuit state equations. The closed loop transfer function of a buck converter is obtained after its dynamic characteristics are determined using small signal analysis (Wu ve ark., 2015 ; Prabhala ve ark., 2016) In this study, dynamic model of the buck converter at continuous current mode was first obtained using small signal state-space method. Afterwards, LQR and LQI controllers were designed for the buck converter to compare dynamic responses of the controllers to different system conditions (changing reference voltage and output load change).

2.Circuit Analysis of DC-DC Buck Converter

The structure of a simplified buck DC-DC converter is shown in Figure 1. Inductor current and capacitor voltage of the buck converter as state variables were denoted as i_L and V_C, respectively.

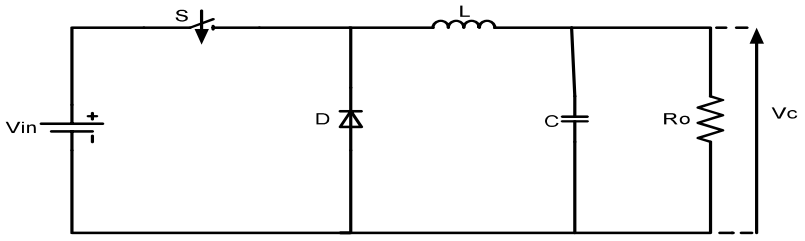


Figure 1. Buck type DC-DC converter circuit

The equivalent circuit in Fig. 2 is obtained when (S) switch is on.

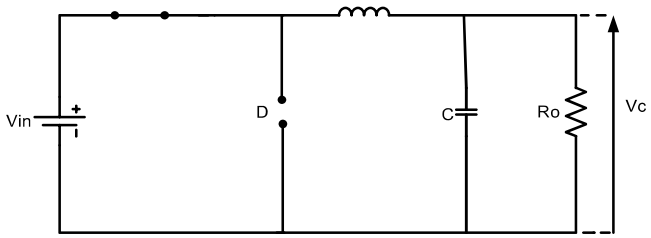


Fig. 2. The equivalent circuit of the buck converter when the switch is on.

When S switch is on, the equations defining the dynamic behavior of the circuit are given in Eqs. 1 and 2.

$$\frac{di_L}{dt} = \frac{V_{in} - V_c}{L} \tag{1}$$

$$\frac{dV_o}{dt} = \frac{i_L}{C} - \frac{V_c}{RC} \tag{2}$$

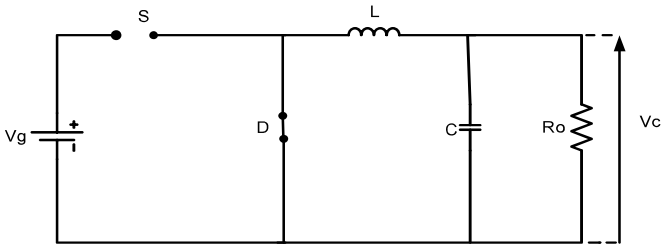


Fig. 3. The equivalent circuit of the buck converter when the switch is off.

The equivalent circuit in Fig. 3 is obtained when (S) switch is off. The equations defining the dynamic behavior of the circuit when S switch is off are given in Eqs.3 and 4.

$$\frac{di_L}{dt} = -\frac{V_c}{L} \tag{3}$$

$$\frac{dV_c}{dt} = \frac{i_L}{C} - \frac{V_c}{RC} \tag{4}$$

The relationship between the output and input voltage at steady state for an ideal buck type DC-DC converter is defined as given in Eq. 5.

$$V_c = DV_{in} \tag{5}$$

The duty ratio for a buck DC-DC converter is given in Eq. 6.

$$D = \frac{T_{on}}{T} = \frac{T_{on}}{T_{on} + T_{off}} \tag{6}$$

T_{on} denotes "Logic 1, On" duration of PWM, T_{off} is the "Logic 0, Off" duration of PWM, T represents the period of PWM, and D defines the duty ratio. For average state-space model, $x(t)$ state variables and state equations with $u(t)$ being control input signal can be given in vector matrix form using differential equations as shown in Eqs. 7-8 (Mohan ve ark., 2002; Erickson, 1997).

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) \tag{7}$$

$$y(t) = Cx(t) \tag{8}$$

When the switch is off (S_{on}=D), the average state-space- form of the buck converter can be written as shown in Eq. 9.

$$\begin{pmatrix} \frac{di_L(t)}{dt} \\ \frac{dV_c(t)}{dt} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} i_L(t) \\ V_c(t) \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} \left(V_{in}(t)\right) \quad (9)$$

When the switch is off (Soff =1-D), the average space-state form of the buck converter can be written as shown in Eq. 10.

$$\begin{pmatrix} \frac{di_L(t)}{dt} \\ \frac{dV_c(t)}{dt} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} i_L(t) \\ V_c(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \left(V_{in}(t)\right) \quad (10)$$

When the switch is both on and off, the average value coefficients of average space-state model of the buck DC-DC converter are given in Eqs. 11-12-13

$$A = A_1 S_{on} + A_2 S_{off} \quad (11)$$

$$B = B_1 S_{on} + B_2 S_{off} \quad (12)$$

$$C = C_1 S_{on} + C_2 S_{off} \quad (13)$$

Average state-space model of the buck DC-DC converter using *A, B, C* coefficients are given in Eqs. 14-15.

$$\begin{pmatrix} \frac{di_L(t)}{dt} \\ \frac{dV_c(t)}{dt} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} \begin{pmatrix} i_L(t) \\ V_c(t) \end{pmatrix} + \begin{pmatrix} \frac{d}{L} \\ 0 \end{pmatrix} \left(V_{in}(t)\right) \quad (14)$$

$$y(t)=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}i_L(t)\\ V_c(t)\end{pmatrix} \tag{15}$$

The variations at operating point for the buck DC-DC converter can be written using small signal model $\left(\tilde{d},\tilde{V}_c,\tilde{V}_g,\tilde{i}_L\right)$ as follows:

$$d=D+\tilde{d};V_c=V_c+\tilde{V}_c;\mathrm{V}_g=V_g+\tilde{V}_g;i_L=i_{_L}+\tilde{i}_L$$

The state equation of $\tilde{i}_L$ current when the switch is both on and off is given in Eq. 16.

$$\frac{d\,\tilde{i}_L(t)}{dt}=d\left[\frac{1}{L}\left(V_{in}+\tilde{V}_{in}-V_c-\tilde{V}_c\right)\right]+(1-d)\left[\frac{1}{L}\left(-V_c-\tilde{V}_c\right)\right] \tag{16}$$

The state equation of $\tilde{V}_c$ voltage when the switch is both on and off is given in Eq. 17.

$$\frac{d\tilde{V}_c(t)}{dt}=\frac{d}{C}\left[\left(I_L+\tilde{i}_L-\frac{V_c}{R}-\frac{\tilde{V}_c}{R}\right)\right]+\frac{(1-d)}{C}\left[\left(I_L+\tilde{i}_L-\frac{V_c}{R}-\frac{\tilde{V}_c}{R}\right)\right] \tag{17}$$

When the switch is both on and off, the small signal model equation of $\tilde{i}_L$ current and $\tilde{V}_c$ voltage is given in Eq. 18.

$$\begin{pmatrix}\frac{d\tilde{i}_L(t)}{dt}\\ \frac{d\tilde{V}_c(t)}{dt}\end{pmatrix}=\begin{pmatrix}0 & -\frac{1}{L}\\ \frac{1}{C} & -\frac{1}{RC}\end{pmatrix}\begin{pmatrix}\tilde{i}_L(t)\\ \tilde{V}_c(t)\end{pmatrix}+\begin{pmatrix}\frac{\tilde{V}_{in}}{L}\\ 0\end{pmatrix}\tilde{d}+\begin{pmatrix}\frac{D}{L}\\ 0\end{pmatrix}\tilde{V}_{in} \tag{18}$$

If superposition theorem is applied to the output voltage of the buck converter,

$$\tilde{V}_c(s)=T_{p1}(s).\tilde{d}(\mathrm{s})+T_{p2}(s)\tilde{V}_g(s) \tag{19}$$

$\tilde{V}_{in}(s) \approx 0$, Eqs. 20-21 are obtained when Laplace transformation of Eq. 18 is used.

$$s\tilde{i}_L(s) = -\frac{1}{L}\tilde{V}_c(s) + \frac{V_{in}}{L}\tilde{d}(s) + \frac{D}{L}\tilde{V}_{in}(s) \quad (20)$$

$$s\tilde{V}_c(s) = \frac{1}{C}\tilde{i}_L(s) - \left(\frac{1}{RC}\right)\tilde{V}_c(s) \quad (21)$$

If $\tilde{i}_L(s)$ is omitted from Eq. 20 and added to Eq. 21, the switching ratio transfer function based on output voltage of the buck converter using small signal model is obtained as shown in Eq. 22 (Cheng K.W.E. 2002).

$$\frac{\tilde{V}_c(s)}{\tilde{d}(s)} = \frac{V_{in}}{LCs^2 + \frac{L}{R}s + 1} \quad (22)$$

3.LQR Control

The term “linear quadratic” function represents the dynamics and quadratic cost function of a linear system. LQR design is based on the selection of feedback gain, thus minimizing J, the cost function, and making the selected gain optimal for the related cost function. The system must be defined using a state-space model for LQR design. The state-space form of the system is given in Eqs. 23-24.

$$\dot{x} = Ax + Bu \quad (23)$$

$$y = Cx + Du \quad (24)$$

The block diagram for the state-space LQR control system is given in Fig. 4.

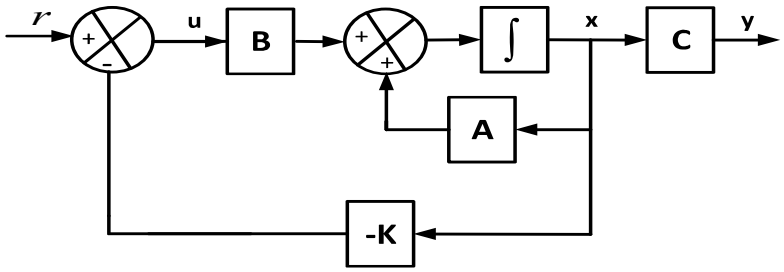


Fig. 4. The block diagram for state-space model LQR control.

The cost function, J, for optimal control is defined in Eq. 25.

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \tag{25}$$

Here, Q and R are weight matrices and can be symmetric and positively defined. It is necessary that Q matrix be completely positive and symmetric for a quadratic form. Q matrix is selected as a diagonal matrix because it can always be replaced by a symmetric Q matrix in quadratic control. Feedback control function is defined as follows:

The value of K is calculated as follows:

$$K = R^{-1} B^T P \tag{27}$$

The continuous time algebraic Riccati equation can be solved to calculate *P*.

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{28}$$

Feedback control function is added to J, which is the cost function:

$$J = \int_0^\infty (x^T Q x + (-Kx)^T R(-Kx))dt \tag{29}$$

$$J = \int_0^\infty x^T (Q + K^T R x) x dt \tag{30}$$

After K matrix is found, state-space equations of the closed loop system are given in Eq.31 (Karanjkar ve ark.,2014).

$$A_{closed-loop} = A - BK, B_{closed-loop} = B, C_{closed-loop} = C - DK, D_{closed-loop} = D \tag{31}$$

3.LQI Controller

LQR controller creates a systematical calculation model for state-feedback control gain matrix. The integral effect in a control system decreases steady-state error for a more robust system response under changing conditions (changing reference, voltage, and load). LQI controller is obtained by adding integral effect to the LQR controller. The block diagram for space state model LQI control system is given in Fig. 5.

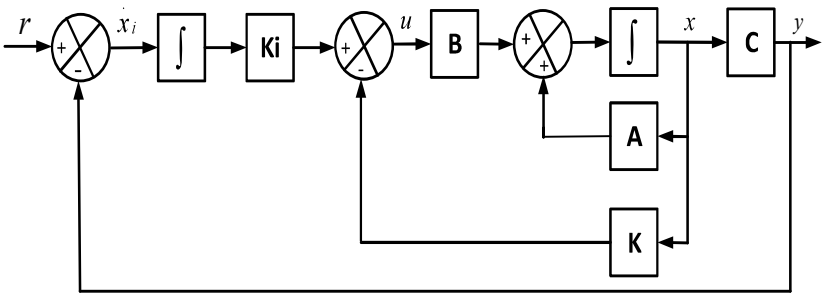


Fig. 5. The block diagram for state-space model LQI control system

The average state-space model obtained by adding integral effect to LQR control is given in Eqs. 32-34.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} u \tag{32}$$

$$y = \begin{pmatrix} c_{11} & c_{12} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{33}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_i \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ -c_{11} & -c_{12} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_i \end{pmatrix} + \begin{pmatrix} b_{11} \\ b_{21} \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r \tag{34}$$

The newly formed state-space matrices are given in Eqs. 35-37.

$$A_{new} = \begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix} \tag{35}$$

$$B_{new} = \begin{pmatrix} B \\ 0 \end{pmatrix} \tag{36}$$

$$C_{new} = \begin{pmatrix} C & 0 \end{pmatrix} \tag{37}$$

K: State-feedback gain can be defined as shown in Eq. 38

$$K = \begin{pmatrix} k_{i_L} & k_{V_0} \end{pmatrix} \tag{38}$$

When K_i integral effect error gain, feedback gain of the LQI controller can be defined as shown in Eq. 39.

$$K_{new} = \begin{pmatrix} K & -k_i \end{pmatrix} \tag{39}$$

4.Simulation Studies

Except uncontrollable system states, LQR control makes the designed system steady and robust.(Uchôa Reis ve ark.,2011) In this study, small signal model of the buck converter was taken into account in the LQR and LQI controller design. The circuit parameters of the buck converter at continuous current mode are given in Table 1.

Parameters	Symbol	Value
Input voltage	V_{in}	20.8 V
Inductance	L	1 mH
Capacitance	C	100 µF
Resistance	R	7.55 Ω
Output voltage	$V_c=V_0^{des}$	16.64 V
Maximum Power	P_{max}	36.8 W
Switching Frequency	f_s	80 Khz
Switching Ratio	D	0.8
Reference Current	i_L^{des}	2.2 A

Table 1. The circuit parameters of the buck converter at continuous current mode.

Using values in Table 1, the switching ratio transfer function based on output voltage is given in Eq. 40.

$$\frac{\tilde{V}_c(s)}{\tilde{d}(s)} = \frac{20.8}{0.0000001s^2 + 0.0001324s + 1} \tag{40}$$

This transfer function must be transformed into a state-space form for LQR controller design. Transfer function can be transformed into a state-space form using MATLAB *tf2*ss, which yielded A, B, C and D matrices given in Eq. 41.

$$A_{new}=\begin{pmatrix}-1324 & -2441.4\\ 4096 & 0\end{pmatrix}, B=\begin{pmatrix}256\\ 0\end{pmatrix}, C=\begin{pmatrix}0 & 198.3643\end{pmatrix}, D=0 \quad (41)$$

The selection of Q and R matrices determines the optimization in an optimal control system design. The components of Q and R matrices are selected by a trial and error method to find the best controller response. Optimal Q and R matrices obtained by a trial and error method using MATLAB simulation program for LQR controller design are given in Eq. 42.

$$Q=\begin{pmatrix}2000 & 0\\ 0 & 0\end{pmatrix}, R=1 \quad (42)$$

K, which is the gain matrix, was determined using the above-mentioned values of Q and R matrices thanks to LQR (A, B, Q and R) instructions in MATLAB simulation program.

$$K=\begin{pmatrix}39.8475 & 0\end{pmatrix} \quad (43)$$

In a LQI controller design, A_{new}, B_{new} and C_{new} state matrices and K_{new} state feedback gain matrix are calculated using A, B and C state matrices and K state feedback gain matrix. A_{new}, B_{new} and C_{new} state matrices and K_{new} state feedback gain matrix are given in Eq. 44-46.

$$A_{new}=\begin{pmatrix}-1324 & -2441.4 & 0\\ 4096 & 0 & 0\\ 0 & -198.3643 & 0\end{pmatrix} \quad (44)$$

$$B_{new}=\begin{pmatrix}256\\ 0\\ 0\end{pmatrix} \quad (45)$$

$$C_{new}=\begin{pmatrix}0 & 198.3643 & 0\end{pmatrix} \quad (46)$$

Optimal Q and R matrices obtained by a trial and error method using MATLAB simulation program for LQI controller design are given in Eq. 47.

$$Q=\begin{pmatrix}3000 & 0 & 0\\ 0 & 250 & 0\\ 0 & 0 & 100\end{pmatrix}, R=1 \tag{47}$$

K, which is the gain matrix, was determined using the above-mentioned values of Q and R matrices thanks to LQR (A, B, Q and R) instructions in MATLAB simulation program.

$$K_{new}=\begin{pmatrix}52.7871 & 10.3903 & -10\end{pmatrix} \tag{48}$$

Control rule for LQI controller is given in Eq. 49.

$$d=D-k_{i_L}(i_L-i_L^{des})-k_{v_o}(V_o-V_o^{des})-k_i\int_0^t(V_0-V_o^{des})dt \tag{49}$$

$V_0^{des}=V_c$, i_L^{des} and D values are given in Table 1. Control rule block diagram for LQI controller is shown in Fig. 6.

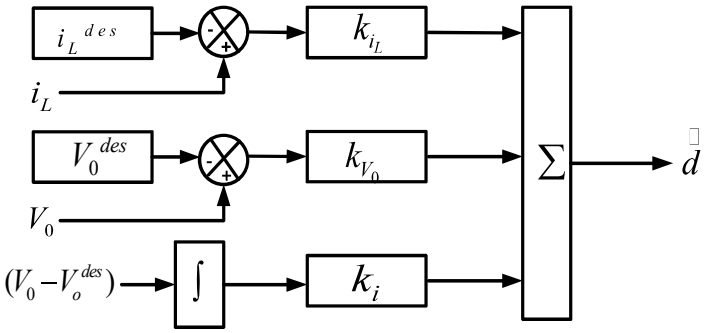
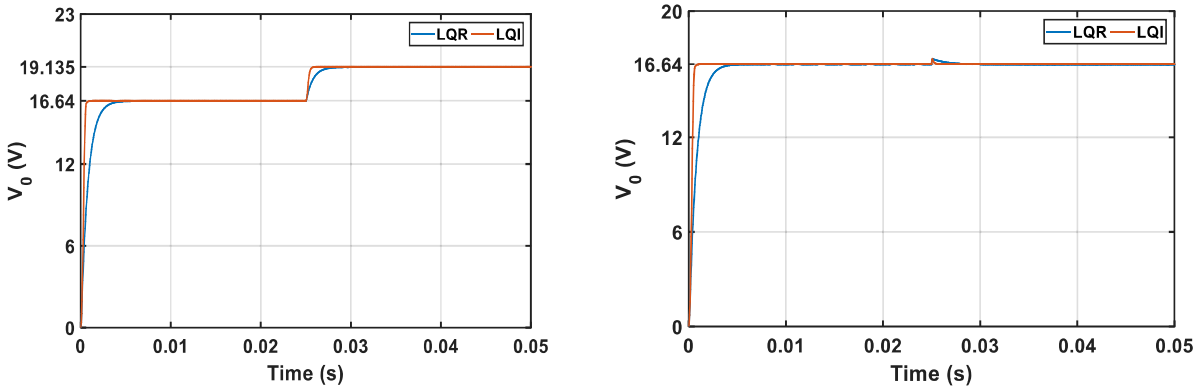


Fig. 6. Control rule block diagram for LQI controller

LQR and LQI controllers were designed in MATLAB using the circuit model of the buck converter. During the simulation studies, the reference input tracking performances of LQR and LQI controllers, which are given step functions, were analyzed. For this purpose, the reference voltage was increased from 16.64 V to 19.135 V at t=0.025 sec. Dynamic responses of LQR and LQI controllers analyzed for reference voltage variations are shown in Fig. 7-a.



(a) (b)

Fig. 7. a) Dynamic responses of LQR and LQI controllers for reference voltage variations.
b) Dynamic responses of LQR and LQI controllers for load variation.

When dynamic responses of controllers against different reference variations shown in Fig. 7-a is analyzed, it can be noted that LQI controller has a better response rate compared to LQR controller. In addition, LQI controller eliminated steady state errors in different reference voltages. Secondly, the load value of the buck DC-DC converter was increased from 7.55 Ω to 15.1 Ω at t=0.025 sec. in order to analyze dynamic responses of LQR and LQI controllers for load variation, and the results are shown in Fig. 7-b.

When dynamic responses of controllers against load variation shown in Fig. 7-b is analyzed, it can be noted that LQI controller responded effectively against load variation without any steady-state errors.

5.Conclusion

In this study, LQR and LQI controllers were designed in Matlab/Simulink simulation program for system control by obtaining small signal model of a buck DC-DC converter. LQR controller only provides modulating gain. Therefore, LQI controller was designed by adding integral effect to LQR control in order to eliminate steady-state error, improve response rate and obtain a more effective controller response to load variation. Two different control scenarios (reference voltage tracking and load variation) were created in order to analyze the dynamic performances of the controllers. The simulation studies demonstrate that LQI controller displayed a more effective and robust dynamic performance compared to LQR controller in terms of reference voltage tracking and load variation.

References

Calderon A. J., Vinagre B., V. Feliu, 2006, Fractional order control strategies for power electronic buck converters, Signal Processing, ELSEVIER 2803–28190165-1684.

Cheng K.W.E., 2002. Classical switched mode and resonant power converters, Hong Kong Polytechnic University, ISBN 962-367-364-7.

Dekker M.,K.K.Sum: 1984.Switch mode power conversion, Inc., pp. 134-188.

Erickson R. W., 1997, Fundamentals of Power Electronics New York, Chapman and Hall.

Karanjkar D.S., Chatterji S., Kumar Amod, 2014. Design and Implementation of a Linear Quadratic Regulator Based Maximum Power Point Tracker for Solar Photo-Voltaic System, International Journal of Hybrid Information Technology.Vol.7, No.1, pp.167-182.

Mohan N., Unlead T. M., Robbins W.P., 2002, Power Electronics, John Wiley & Sons Ltd., England, 185-191.

Prabhala V. A. K., Fajri P., Gouribhatla V. S. P., Baddipadiga B. P., Ferdowsi M., 2016. A DC-DC converter with high voltage gain ve two input boost stages, IEEE Trans. Power Electronics, Vol. 31, no. 6, pp. 4206– 4215.

Uchôa Reis F. E., Torrico-Bascopé R.P., Costa M.V.S., 2011, LQR control with integral action applied to a high gain step-up DC-DC converter, Power Electronics Conference (COBEP), Brazilian.

Wu K. C.,Wu H. H., ve Wei C. L., Dec. 2015, Analysis and design of mixed-mode operation for noninverting buck-boost DC-DC converters, IEEE Trans. Circuits ve Systems II-Express Briefs, Vol. 62, no. 12, pp. 1194–1198.